Zustandsmodell

kontinuierlich

$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{m}(t), t)$ Zustand

$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{n}(t), t)$ Messung

diskret

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{m}_k) = \int_{t_k}^{t_{k+1}} \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), \mathbf{m}(t), t)\mathrm{d}t + \mathbf{x}_k \approx T\mathbf{f}(\mathbf{x}(t_k), \mathbf{u}(t_k), \mathbf{m}(t_k), t_k) + \mathbf{x}_k$$

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{n}_k) = \mathbf{h}(\mathbf{x}(t_k), \mathbf{u}(t_k), \mathbf{m}(t_k), t_k)$$

Signale

$\mathbf{u} \sim \mathcal{N}(\tilde{\mathbf{u}}, \mathbf{U}\,[tu^2])$ Systemeingang

$\mathbf{m} \sim \mathcal{N}(\mathbf{0}, \mathbf{M}\,[tm^2])$ Prozeßrauschen

$\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \mathbf{N}\,[tn^2])$ Meßrauschen

$\mathbf{u}_k \sim \mathcal{N}(\tilde{\mathbf{u}}, \mathbf{U}_k\,[u^2])$ $\quad \mathbf{u}_k = \mathbf{u}(t_k)$ $\quad \mathbf{U}_k \approx T^{-1}\mathbf{U}(t_k)$

$\mathbf{m}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{M}_k\,[m_k^2])$ $\quad \mathbf{m}_k = \int_{t_k}^{t_{k+1}} \mathbf{m}(t)\mathrm{d}t \approx T\mathbf{m}(t_k)$ $\quad \mathbf{M}_k \approx T\mathbf{M}(t_k)$

$\mathbf{n}_k \sim \mathcal{N}(\mathbf{0}, \mathbf{N}_k\,[n^2])$ $\quad \mathbf{n}_k = \mathbf{n}(t_k)$ $\quad \mathbf{N}_k \approx T^{-1}\mathbf{N}(t_k)$

$$\mathbf{B}_k \cdot \mathbf{U}_k \cdot \mathbf{B}_k^{\mathrm{T}} = \int_{t_k}^{t_{k+1}} \mathbf{A}'(t, t_{k+1}) \cdot \mathbf{B} \cdot \mathbf{U} \cdot \mathbf{B}^{\mathrm{T}} \cdot \mathbf{A}'(t, t_{k+1})^{\mathrm{T}}\mathrm{d}t$$

$$\mathbf{E}_k \cdot \mathbf{M}_k \cdot \mathbf{E}_k^{\mathrm{T}} = \int_{t_k}^{t_{k+1}} \mathbf{A}'(t, t_{k+1}) \cdot \mathbf{E} \cdot \mathbf{M} \cdot \mathbf{E}^{\mathrm{T}} \cdot \mathbf{A}'(t, t_{k+1})^{\mathrm{T}}\mathrm{d}t$$

Linearisierung (EKF)

$\dot{\mathbf{x}}(t) = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u} + \mathbf{E} \cdot \mathbf{m} + \dot{\mathbf{x}}_{\mathbf{f}}$ $\quad [\frac{x}{t}]$

$\mathbf{y}(t) = \mathbf{C} \cdot \mathbf{x} + \mathbf{D} \cdot \mathbf{u} + \mathbf{F} \cdot \mathbf{n} + \mathbf{y}_{\mathbf{h}}$ $\quad [y]$

$\mathbf{A}(t) = \frac{\partial \mathbf{f}}{\partial \mathbf{x}^{\mathrm{T}}}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t)$ $\quad [\frac{1}{t}]$

$\mathbf{B}(t) = \frac{\partial \mathbf{f}}{\partial \mathbf{u}^{\mathrm{T}}}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t)$ $\quad [\frac{x}{tu}]$

$\mathbf{C}(t) = \frac{\partial \mathbf{h}}{\partial \mathbf{x}^{\mathrm{T}}}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t)$ $\quad [\frac{y}{x}]$

$\mathbf{D}(t) = \frac{\partial \mathbf{h}}{\partial \mathbf{u}^{\mathrm{T}}}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t)$ $\quad [\frac{y}{u}]$

$\mathbf{E}(t) = \frac{\partial \mathbf{f}}{\partial \mathbf{m}^{\mathrm{T}}}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t)$ $\quad [\frac{x}{tm}]$

$\mathbf{F}(t) = \frac{\partial \mathbf{h}}{\partial \mathbf{n}^{\mathrm{T}}}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t)$ $\quad [\frac{y}{n}]$

$\dot{\mathbf{x}}_{\mathbf{f}}(t) = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) - \mathbf{A} \cdot \mathbf{x} - \mathbf{B} \cdot \mathbf{u}$

$\mathbf{y}_{\mathbf{h}}(t) = \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t) - \mathbf{C} \cdot \mathbf{x} - \mathbf{D} \cdot \mathbf{u}$

$\mathbf{x}_{k+1} = \mathbf{A}_k \cdot \mathbf{x}_k + \mathbf{B}_k \cdot \mathbf{u}_k + \mathbf{E}_k \cdot \mathbf{m}_k + \mathbf{x}_{\mathbf{f}k}$ $\quad = \mathbf{x}(t_{k+1})$ $\quad [x]$

$\mathbf{y}_k = \mathbf{C}_k \cdot \mathbf{x}_k + \mathbf{D}_k \cdot \mathbf{u}_k + \mathbf{F}_k \cdot \mathbf{n}_k + \mathbf{y}_{\mathbf{h}k}$ $\quad = \mathbf{y}(t_k)$ $\quad [y]$

$\mathbf{A}_k = \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}^{\mathrm{T}}}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0}) = \mathbf{A}'(t_k, t_{k+1}) := \exp\left(\int_{t_k}^{t_{k+1}} \mathbf{A}\mathrm{d}t\right)$ $\quad \approx T\mathbf{A}(t_k) + \mathbf{1}$ $\quad [1]$

$\mathbf{B}_k = \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}^{\mathrm{T}}}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0})$ $\quad \approx T\mathbf{B}(t_k)$ $\quad [\frac{x}{u}]$

$\mathbf{B}_k \cdot \mathbf{u}_k = \int_{t_k}^{t_{k+1}} \mathbf{A}'(t, t_{k+1}) \cdot \mathbf{B} \cdot \mathbf{u}\mathrm{d}t$

$\mathbf{C}_k^+ = \frac{\partial \mathbf{h}_k}{\partial \mathbf{x}^{\mathrm{T}}}(\mathbf{x}_k^+, \mathbf{u}_{k+1}, \mathbf{0})$ $\quad = \mathbf{C}(t_k)$ $\quad [\frac{y}{x}]$

$\mathbf{D}_k^+ = \frac{\partial \mathbf{h}_k}{\partial \mathbf{u}^{\mathrm{T}}}(\mathbf{x}_k^+, \mathbf{u}_{k+1}, \mathbf{0})$ $\quad = \mathbf{D}(t_k)$ $\quad [\frac{y}{u}]$

$\mathbf{E}_k = \frac{\partial \mathbf{f}_k}{\partial \mathbf{m}^{\mathrm{T}}}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0})$ $\quad \approx \mathbf{E}(t_k)$ $\quad [\frac{x}{m_k}]$

$\mathbf{E}_k \cdot \mathbf{m}_k = \int_{t_k}^{t_{k+1}} \mathbf{A}'(t, t_{k+1}) \cdot \mathbf{E} \cdot \mathbf{m}\mathrm{d}t$

$\mathbf{F}_k^+ = \frac{\partial \mathbf{h}_k}{\partial \mathbf{n}^{\mathrm{T}}}(\mathbf{x}_k^+, \mathbf{u}_{k+1}, \mathbf{0})$ $\quad = \mathbf{F}(t_k)$ $\quad [\frac{y}{n}]$

$\mathbf{x}_{\mathbf{f}k} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0}) - \mathbf{A}_k \cdot \mathbf{x}_k - \mathbf{B}_k \cdot \mathbf{u}_k = \int_{t_k}^{t_{k+1}} \mathbf{A}'(t, t_{k+1}) \cdot \dot{\mathbf{x}}_{\mathbf{f}}(t)\mathrm{d}t$ $\quad \approx T\dot{\mathbf{x}}_{\mathbf{f}}(t_k)$

$\mathbf{y}_{\mathbf{h}k} = \mathbf{h}_k(\mathbf{x}_k^+, \mathbf{u}_{k+1}, \mathbf{0}) - \mathbf{C}_k \cdot \mathbf{x}_k - \mathbf{D}_k \cdot \mathbf{u}_k$ $\quad = \mathbf{y}_{\mathbf{h}}(t_k)$

Kalman-Bucy-Filter

$\dot{\mathbf{x}}^+ = \mathbf{f}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t)$

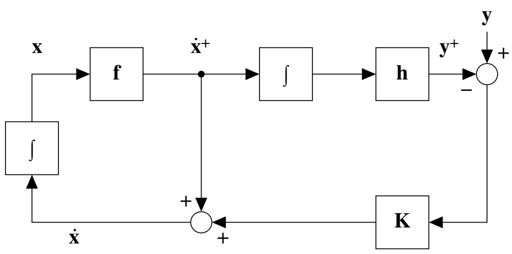
$\mathbf{y}^+ = \mathbf{h}(\mathbf{x}, \mathbf{u}, \mathbf{0}, t)$

$\dot{\mathbf{X}}^+ = \mathbf{A} \cdot \mathbf{X} + \mathbf{X} \cdot \mathbf{A}^{\mathrm{T}} + \mathbf{B} \cdot \mathbf{U} \cdot \mathbf{B}^{\mathrm{T}} + \mathbf{E} \cdot \mathbf{M} \cdot \mathbf{E}^{\mathrm{T}}$

$\mathbf{K} = \mathbf{X} \cdot \mathbf{C}^{\mathrm{T}} \cdot \left(\mathbf{D} \cdot \mathbf{U} \cdot \mathbf{D}^{\mathrm{T}} + \mathbf{F} \cdot \mathbf{N} \cdot \mathbf{F}^{\mathrm{T}}\right)^{-1}$ $\quad [\frac{x}{ty}]$

$\dot{\mathbf{x}} = \dot{\mathbf{x}}^+ + \mathbf{K} \cdot (\mathbf{y} - \mathbf{y}^+)$

$\dot{\mathbf{X}} = \dot{\mathbf{X}}^+ - \mathbf{K} \cdot \mathbf{C} \cdot \mathbf{X}$



Kalman-Filter

$\mathbf{x}_{k+1}^+ = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k, \mathbf{0})$ Zustandsvorhersage

$\mathbf{y}_{k+1}^+ = \mathbf{h}_{k+1}(\mathbf{x}_{k+1}^+, \mathbf{u}_{k+1}, \mathbf{0})$ Meßvorhersage

$\mathbf{X}_{k+1}^+ = \mathbf{A}_k \cdot \mathbf{X}_k \cdot \mathbf{A}_k^{\mathrm{T}} + \mathbf{B}_k \cdot \mathbf{U}_k \cdot \mathbf{B}_k^{\mathrm{T}} + \mathbf{E}_k \cdot \mathbf{M}_k \cdot \mathbf{E}_k^{\mathrm{T}}$ Vorhersagefehler

$\mathbf{Y}_{k+1}^+ = \mathbf{C}_{k+1}^+ \cdot \mathbf{X}_{k+1}^+ \cdot \mathbf{C}_{k+1}^{+\mathrm{T}} + \mathbf{D}_{k+1}^+ \cdot \mathbf{U}_{k+1} \cdot \mathbf{D}_{k+1}^{+\mathrm{T}} + \mathbf{F}_{k+1}^+ \cdot \mathbf{N}_{k+1} \cdot \mathbf{F}_{k+1}^{+\mathrm{T}}$

$\mathbf{K}_{k+1} = \mathbf{X}_{k+1}^+ \cdot \mathbf{C}_{k+1}^{+\mathrm{T}} \cdot \mathbf{Y}_{k+1}^{+-1}$ $\quad [\frac{x}{y}]$ Kalman-Gain

$\mathbf{x}_{k+1} = \mathbf{x}_{k+1}^+ + \mathbf{K}_{k+1} \cdot \left(\mathbf{y}_{k+1} - \mathbf{y}_{k+1}^+\right)$ Neuer Zustand

$\mathbf{X}_{k+1} = \left(\mathbf{1} - \mathbf{K}_{k+1} \cdot \mathbf{C}_{k+1}^+\right) \cdot \mathbf{X}_{k+1}^+$ Neue Zustandsvarianz, Joseph-Form:

$$= \left(\mathbf{1} - \mathbf{K}_{k+1} \cdot \mathbf{C}_{k+1}^+\right) \cdot \mathbf{X}_{k+1}^+ \cdot \left(\mathbf{1} - \mathbf{K}_{k+1} \cdot \mathbf{C}_{k+1}^+\right)^{\mathrm{T}} + \mathbf{K}_{k+1} \cdot \mathbf{N}_{k+1} \cdot \mathbf{K}_{k+1}^{\mathrm{T}}$$