

## Matrizen

Matrix

$$\mathbf{A} = (a_{ij}) = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \cdots & a_{ij} & \cdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

Determinante

$$\det(\mathbf{A}) = |\mathbf{A}| = |\mathbf{A}^T| = \sum_{i=1}^m a_{ij} A_{ij} = \sum_{j=1}^n a_{ij} A_{ij}$$

inverse

$$\mathbf{A}^{-1} = \frac{1}{|\mathbf{A}|} \begin{pmatrix} A_{11} & \cdots & A_{n1} \\ \cdots & A_{ij} & \cdots \\ A_{1m} & \cdots & A_{nm} \end{pmatrix}$$

Einheitsmatrix

$$\mathbf{1} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix} = \mathbf{A} \cdot \mathbf{A}^{-1}$$

Adjunkte

$$A_{ij} = (-1)^{i+j} \begin{pmatrix} a_{11} & \cdots & a_{i(j-1)} & a_{1(j+1)} & \cdots & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{(i-1)1} & \cdots & a_{(i-1)(j-1)} & a_{(i-1)(j+1)} & \cdots & a_{(i-1)n} \\ a_{(i+1)1} & \cdots & a_{(i+1)(j-1)} & a_{(i+1)(j+1)} & \cdots & a_{(i+1)n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & \cdots & a_{m(j-1)} & a_{m(j+1)} & \cdots & a_{mn} \end{pmatrix}$$

Transposition

$$\mathbf{A}^T = (a_{ji}) \quad (\mathbf{A} \cdot \mathbf{B})^T = \mathbf{B}^T \cdot \mathbf{A}^T \quad (\mathbf{A} \cdot \mathbf{B})^{-1} = \mathbf{B}^{-1} \cdot \mathbf{A}^{-1}$$

Rechenregeln

$$\begin{aligned} |(\mathbf{a}_1 + \mathbf{a}_2 \quad \mathbf{a}_3)| &= |(\mathbf{a}_1 \quad \mathbf{a}_3)| + |(\mathbf{a}_2 \quad \mathbf{a}_3)| & \lambda |(\mathbf{a}_1 \quad \mathbf{a}_2)| &= |(\lambda \mathbf{a}_1 \quad \mathbf{a}_2)| & |\mathbf{A} \cdot \mathbf{B}| &= |\mathbf{A}| |\mathbf{B}| \\ |(\mathbf{a}_1 \quad \mathbf{a}_2)| &= |(\mathbf{a}_1 \quad (\mathbf{a}_2 + \lambda \mathbf{a}_1))| & |(\mathbf{a}_1 \quad \mathbf{a}_2 \quad \mathbf{a}_3)| &= -|(\mathbf{a}_2 \quad \mathbf{a}_1 \quad \mathbf{a}_3)| & |(\mathbf{a}_1 \quad \mathbf{a}_1 \quad \mathbf{a}_2)| &= 0 \end{aligned}$$

Eigenwerte  $\lambda_i$

$$P(\lambda) = \det(\mathbf{A} - \lambda \mathbf{1}) = \prod_{i=0}^k (\lambda - \lambda_i)^{\alpha_i} = 0$$

Eigenvektoren  $\lambda_{ij}$  zu  $\lambda_i$

$$(\mathbf{A} - \lambda_i \mathbf{1}) \cdot \lambda_{ij} = \mathbf{0} \quad 0 \leq j \leq \beta_i$$

Vielfachheiten

$$1 \leq \beta_i \leq \alpha_i$$

$\alpha_i$  algebraische Vf.  
 $\beta_i$  geometrische Vf.

Orthogonalisieren

$$\mathbf{b}_j = \lambda_{ij} - \sum_{q=1}^{j-1} \frac{\lambda_{ij} \cdot \mathbf{b}_q}{\mathbf{b}_q^2} \mathbf{b}_q \quad \mathbf{B} = (\mathbf{b}_j)$$

Definitheit

$$\begin{aligned} \forall \mathbf{x} \neq \mathbf{0}: \bar{\mathbf{x}}^T \cdot \mathbf{A} \cdot \mathbf{x} > 0 &\Leftrightarrow \forall i: \lambda_i > 0 &\Leftrightarrow \mathbf{A} \text{ ist positiv definit} \\ \forall \mathbf{x} \neq \mathbf{0}: \bar{\mathbf{x}}^T \cdot \mathbf{A} \cdot \mathbf{x} \geq 0 &\Leftrightarrow \forall i: \lambda_i \geq 0 &\Leftrightarrow \mathbf{A} \text{ ist positiv semidefinit} \\ \forall \mathbf{x} \neq \mathbf{0}: \bar{\mathbf{x}}^T \cdot \mathbf{A} \cdot \mathbf{x} < 0 &\Leftrightarrow \forall i: \lambda_i < 0 &\Leftrightarrow \mathbf{A} \text{ ist negativ definit} \\ \forall \mathbf{x} \neq \mathbf{0}: \bar{\mathbf{x}}^T \cdot \mathbf{A} \cdot \mathbf{x} \leq 0 &\Leftrightarrow \forall i: \lambda_i \leq 0 &\Leftrightarrow \mathbf{A} \text{ ist negativ semidefinit} \end{aligned}$$

sonst:  $\mathbf{A}$  ist indefinit

## Vektor

Spaltenvektor

$$\mathbf{a} = (a_i) = \begin{pmatrix} a_1 \\ \vdots \\ a_m \end{pmatrix}$$

Zeilenvektor

$$\mathbf{a}^T = (a_j) = (a_1 \quad \cdots \quad a_n)$$

Skalarprodukt

$$\mathbf{a}_1 \cdot \mathbf{a}_2 = \mathbf{a}_1^T \cdot \mathbf{a}_2$$

Kreuzprodukt

$$\mathbf{a}_1 \times \mathbf{a}_2 = \begin{pmatrix} a_{12}a_{23} - a_{13}a_{22} \\ a_{13}a_{21} - a_{11}a_{23} \\ a_{11}a_{22} - a_{12}a_{21} \end{pmatrix}$$

Betrag

$$|\mathbf{a}| = \sqrt{\mathbf{a} \cdot \mathbf{a}}$$

Division

$$\frac{\mathbf{a}_1}{\mathbf{a}_2} = \frac{\mathbf{a}_1 \cdot \mathbf{a}_2}{\mathbf{a}_2^2}$$

elementweise

$$\frac{\mathbf{a}_1}{\mathbf{a}_2} = \begin{pmatrix} a_{11} \\ \vdots \\ a_{1m} \end{pmatrix} \cdot \left( \frac{1}{a_{21}} \quad \cdots \quad \frac{1}{a_{2n}} \right)$$

Richtungsableitung

$$\frac{\partial \mathbf{a}_1}{\partial \mathbf{a}_2} = (\mathbf{a}_2 \cdot \nabla) \mathbf{a}_1$$

Jacobi-Matrix

$$\frac{\partial \mathbf{a}_1}{\partial \mathbf{a}_2^T} = \begin{pmatrix} \frac{\partial a_{11}}{\partial a_{21}} & \cdots & \frac{\partial a_{11}}{\partial a_{2n}} \\ \cdots & \frac{\partial a_{1i}}{\partial a_{2j}} & \cdots \\ \frac{\partial a_{1m}}{\partial a_{21}} & \cdots & \frac{\partial a_{1m}}{\partial a_{2n}} \end{pmatrix}$$

## Gleichungssystem (Cramer)

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b} \quad x_i = \frac{|(\mathbf{a}_1 \quad \cdots \quad \mathbf{a}_{i-1} \quad \mathbf{b} \quad \mathbf{a}_{i+1} \quad \cdots \quad \mathbf{a}_n)|}{|\mathbf{A}|}$$

### Quadriken

$$a(\mathbf{x}) = \left( \sum_{i=1}^n a_i x_i \right)^2 + \sum_{i=1}^n d_i x_i + c = (\mathbf{x}^T \cdot \mathbf{A} \cdot \mathbf{x}) + (\mathbf{d}^T \cdot \mathbf{x}) + c = \sum_{i=1}^n (\lambda_i z_i)^2 + c' = 0$$

$$a(\mathbf{x}) = (\mathbf{y}^T \cdot (\mathbf{B}^T \cdot \mathbf{A} \cdot \mathbf{B}) \cdot \mathbf{y}) + (\mathbf{d}^T \cdot \mathbf{B} \cdot \mathbf{y}) + c = (\mathbf{z}^T \cdot (\mathbf{B}^T \cdot \mathbf{A} \cdot \mathbf{B}) \cdot \mathbf{z}) + \left( c - \frac{1}{4} \mathbf{d}^T \cdot \mathbf{A}^{-1} \cdot \mathbf{d} \right) = (\mathbf{z}^T \cdot (\mathbf{B}^T \cdot \mathbf{A} \cdot \mathbf{B}) \cdot \mathbf{z}) + \frac{|\mathbf{Q}|}{|\mathbf{A}|}$$

mit  $a_{ij} = \mathbf{a}_i \cdot \mathbf{a}_j$       $\mathbf{Q} = \begin{pmatrix} \mathbf{A} & \frac{1}{2} \mathbf{d} \\ \frac{1}{2} \mathbf{d}^T & c \end{pmatrix}$

Hauptachsentransformation     Translation/quadratische Ergänzung

$\mathbf{x} = \mathbf{B} \cdot \mathbf{y}$       $z_i = y_i + \frac{\mathbf{d}^T \cdot \mathbf{b}_i}{2\lambda_i}$

### Polynom

$$P_n(x) = \sum_{i=0}^n a_i x^i = a_n \prod_{i=1}^n (x - x_i)^{\alpha_i}$$

Partialbruchzerlegung

$$Q_m(x) = a_m \prod_{i=1}^r (x - x_i)^{\alpha_i} \prod_{j=1}^s (x^2 + p_j x + q_j)^{\beta_j} \quad \sum_{i=1}^r \alpha_i + \sum_{j=1}^s \beta_j = m$$

$$\frac{P_n(x)}{Q_m(x)} = \sum_{i=1}^r \sum_{j=1}^r \frac{A_{i\alpha_j}}{(x-x_i)^{\alpha_j}} + \sum_{i=1}^s \sum_{j=1}^s \frac{M_{i\beta_j} x + N_{i\beta_j}}{(x^2 + p_i x + q_i)^{\beta_j}}$$

### Reihen

$$I = \int_0^{\infty} f(\mathbf{x}, n) dn \neq \infty \quad \Leftrightarrow \quad \sum_{n=0}^{\infty} f(\mathbf{x}, n) \neq \infty$$

Konvergenz

$$\lim_{n \rightarrow \infty} (f(n)) \neq 0 \Rightarrow I = \infty$$

$$\lim_{n \rightarrow \infty} (n f(n)) \neq 0 \Rightarrow I = \infty$$

$$\lim_{n \rightarrow \infty} \left( \left| \frac{f(n+1)}{f(n)} \right| \right) > 1 \Rightarrow I = \infty$$

$$\lim_{n \rightarrow \infty} \left( \sqrt[n]{f(n)} \right) > 1 \Rightarrow I = \infty$$

$$\lim_{n \rightarrow \infty} \left( \frac{f(n)}{g(n)} \right) \notin \{0, \infty\} \quad \wedge \quad \int_0^{\infty} g(n) dn \neq \infty \Rightarrow I \neq \infty$$

$$\lim_{n \rightarrow \infty} \left( \left| \frac{f(n+1)}{f(n)} \right| \right) < 1 \Rightarrow I \neq \infty$$

$$\lim_{n \rightarrow \infty} \left( \sqrt[n]{f(n)} \right) < 1 \Rightarrow I \neq \infty$$

$$\int_0^{\infty} |f(n)| dn \neq \infty \Rightarrow I \neq \infty$$

$$f(n) = (-1)^n |g(n)| \quad \wedge \quad \lim_{n \rightarrow \infty} (g(n)) = 0 \Rightarrow I \neq \infty$$

$$\int_N^{\infty} f(n) dn \leq \sum_{n=N}^{\infty} f(n) = S - \sum_{n=1}^{N-1} f(n) \leq f(N) + \int_N^{\infty} f(n) dn$$

Geometrische Reihe

$$s_n = a \sum_{i=0}^{n-1} q^i = a \frac{1-q^n}{1-q}$$

Taylorreihe

$$T(x) = \sum_{i=0}^n \frac{f^{(i)}(x_0)}{i!} (x - x_0)^i + R_n(x)$$

$$R_n(x) = \int_{x_0}^x \frac{(x-t)^n}{n!} f^{(n+1)}(t) dt = \frac{f^{(i+1)}(x \dots x_0)}{(i+1)!} (x - x_0)^{i+1}$$

$$T(\mathbf{x} + d\mathbf{x}) = \sum_{k=0}^n \frac{d^k f(\mathbf{x})}{k!} + R_n(\mathbf{x})$$

$$R_n(\mathbf{x} + d\mathbf{x}) = \frac{d^{k+1} f(\mathbf{x} \dots (\mathbf{x} + d\mathbf{x}))}{(k+1)!}$$