

Elementarsignale

Signumfunktion

$$\operatorname{sgn}(t) = \begin{cases} -1 & \text{für } t < 0 \\ 0 & \text{für } t = 0 \\ 1 & \text{für } t > 0 \end{cases} = 2\sigma(t) - 1 = 2 \int_{-\infty}^t \delta(\tau) d\tau - 1$$

$$\operatorname{sgn}(n) = \begin{cases} -1 & \text{für } n < 0 \\ 1 & \text{für } n \geq 0 \end{cases} = 2\sigma(n) - 1 = 2 \sum_{\nu=-\infty}^n \delta(\nu) - 1$$

$$\mathcal{F}(\operatorname{sgn}(t)) = \frac{2}{i\omega} \quad \mathcal{L}(\operatorname{sgn}(t)) = \frac{2}{s}$$

Sprungfunktion

$$\sigma(t) = \begin{cases} 0 & \text{für } t < 0 \\ 0,5 & \text{für } t = 0 \\ 1 & \text{für } t > 0 \end{cases} = \frac{1}{2}\operatorname{sgn}(t) + \frac{1}{2} = \int_{-\infty}^t \delta(\tau) d\tau = \int_0^{\infty} \delta(t - \tau) d\tau$$

$$\sigma(n) = \begin{cases} 0 & \text{für } n < 0 \\ 1 & \text{für } n \geq 0 \end{cases} = \frac{1}{2}\operatorname{sgn}(n) + \frac{1}{2} = \sum_{\nu=-\infty}^t \delta(\nu) = \sum_{\nu=0}^{\infty} \delta(n - \nu)$$

$$\mathcal{F}(\sigma(t)) = \frac{1}{i\omega} + \pi\delta(\omega) \quad \mathcal{L}(\sigma(t)) = \frac{1}{s}$$

Rechteckfunktion

$$\Pi_T(t) = \begin{cases} 1 & \text{für } |t| < T/2 \\ 0,5 & \text{für } |t| = T/2 \\ 0 & \text{für } |t| > T/2 \end{cases} = \sigma\left(t + \frac{T}{2}\right) - \sigma\left(t - \frac{T}{2}\right)$$

$$\Pi_M(n) = \begin{cases} 1 & \text{für } |n| \leq (M-1)/2 \\ 0 & \text{für } |n| > (M-1)/2 \end{cases} = \sigma\left(n + \frac{M-1}{2}\right) - \sigma\left(n - \frac{M-1}{2}\right)$$

$$\mathcal{F}(\Pi_T(t)) = T \operatorname{si}\left(\frac{\omega T}{2}\right)$$

$$\mathcal{F}(\Pi_M(n)) = \frac{\sin(M\omega T/2)}{\sin(\omega T/2)}$$

Dreieckfunktion

$$\Lambda_T(t) = \begin{cases} 1 - |t|/T & \text{für } |t| \leq T \\ 0 & \text{für } |t| > T \end{cases} = \frac{1}{T} \Pi_T(t) \star \Pi_T(t) = r_T(t+T) - r_T(t)$$

$$\Lambda_M(n) = \begin{cases} 1 - |n|/M & \text{für } |n| \leq M \\ 0 & \text{für } |n| > M \end{cases} = \frac{1}{M} \Pi_M(n) \star \Pi_M(n)$$

$$\mathcal{F}(\Lambda_T(t)) = T \operatorname{si}\left(\frac{\omega T}{2}\right)^2$$

Rampenfunktion

$$r_T(t) = \begin{cases} 0 & \text{für } t < 0 \\ t/T & \text{für } 0 \leq t \leq T \\ 1 & \text{für } t > T \end{cases} = \frac{1}{T} \int_{-\infty}^t \Pi_T\left(\tau - \frac{T}{2}\right) d\tau$$

$$= \frac{1}{T} \Pi_T\left(t - \frac{T}{2}\right) + \sigma(t - T)$$

$$r_M(n) = \begin{cases} 0 & \text{für } n < 0 \\ n/M & \text{für } 0 \leq n \leq M \\ 1 & \text{für } n > M \end{cases} = \frac{1}{M} \left(\sum_{\nu=-\infty}^n \Pi_M\left(\nu - \frac{M}{2}\right) - 1 \right)$$

$$\mathcal{F}(r_T(t)) = \frac{1}{i\omega} \operatorname{si}\left(\frac{\omega T}{2}\right) \exp(-i\frac{\omega T}{2}) + \pi\delta(\omega)$$

si-Funktion (Spaltfunktion)

$$\operatorname{si}(\omega_0 t) = \frac{\sin(\omega_0 t)}{\omega_0 t}$$

$$\operatorname{si}(\omega_0 n) = \frac{\sin(\omega_0 n T)}{\omega_0 n T}$$

$$\mathcal{F}(\operatorname{si}(\omega_0 t)) = \frac{T}{2} \Pi_{\omega_0 T}\left(\frac{\omega}{2}\right)$$

Deltafunktion

$$\delta(t) = \begin{cases} \infty & \text{für } t = 0 \\ 0 & \text{für } t \neq 0 \end{cases} = \lim_{A \rightarrow 0} \left(\frac{1}{A} \Pi_A(t) \right) = \frac{d\sigma(t)}{dt}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(i\omega t) d\omega = \frac{\Omega}{2\pi} \sum_{k=-\infty}^{\infty} \exp(ik\Omega t)$$

$$\delta(n) = \begin{cases} 1 & \text{für } n = 0 \\ 0 & \text{für } n \neq 0 \end{cases} = \Pi_1(n) = \sigma(n) - \sigma(n-1)$$

$$= \frac{T}{2\pi} \int_x^{x+2\pi/T} \exp(i\omega n T) d\omega$$

$$\mathcal{F}(\delta(t + t_0)) = \exp(i\omega t_0) \quad \mathcal{L}(\delta(t + t_0)) = \exp(st_0)$$

$$\mathcal{F}(\delta(n + n_0)) = \exp(i\omega n_0 T) \quad \mathcal{Z}(\delta(n + n_0)) = z^{n_0}$$

$$u(t + t_0) = u(t) \star \delta(t + t_0) = \int_{-\infty}^{\infty} u(\tau) \delta(t + t_0 - \tau) d\tau$$

$$u(n + n_0) = u(n) \star \delta(n + n_0) = \sum_{\nu=-\infty}^{\infty} u(\nu) \delta(n + n_0 - \nu)$$

Deltakamm

$$\delta_T(t) = \sum_{\tau=-\infty}^{\infty} \delta(t - \tau T)$$

$$\delta_M(n) = \sum_{\nu=-\infty}^{\infty} \delta(n - \nu M)$$

$$\mathcal{F}(\delta_T(t)) = \omega_T \delta_{\omega_T}(\omega)$$

$$\mathcal{Z}(\delta_M(n)) = \sum_{\nu=-\infty}^{\infty} z^{-\nu M}$$

$$u_T(t) = u(t) \star \delta_T(t) = \sum_{k=-\infty}^{\infty} u(t - kT)$$

Signaleigenschaften

Arithmetischer Mittelwert

$$m_u(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} u(t) dt$$

$$m_u(n_1, n_2) = \frac{1}{n_2 - n_1 + 1} \sum_{v=n_1}^{n_2} u(v)$$

$$m_u = m_u(-\infty, \infty) = m_{u_T}(\tau, \tau + T)$$

Gleichrichtmittelwert

$$m_{|u|}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |u(t)| dt$$

$$m_{|u|}(n_1, n_2) = \frac{1}{n_2 - n_1 + 1} \sum_{v=n_1}^{n_2} |u(v)|$$

$$m_{|u|} = m_{|u|}(-\infty, \infty) = m_{|u_T|}(\tau, \tau + T)$$

Effektivwert

$$m_{u^2}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} u(t)^2 dt$$

$$m_{u^2}(n_1, n_2) = \frac{1}{n_2 - n_1 + 1} \sum_{v=n_1}^{n_2} u(v)^2$$

$$U_{\text{eff}} = m_{u^2} = m_{u^2}(-\infty, \infty) = m_{u_T^2}(\tau, \tau + T)$$

Energie

$$W_u(t_1, t_2) = \frac{1}{Z} \int_{t_1}^{t_2} u(t)^2 dt = \frac{1}{Z} r_{uu}(t_1, t_2)(0)$$

$$W_u(n_1, n_2) = \frac{1}{Z} \sum_{v=n_1}^{n_2} u(v)^2 = \frac{1}{Z} r_{uu}(n_1, n_2)(0)$$

$$W_u = W_u(-\infty, \infty) = W_{u_T}(\tau, \tau + T) = \frac{1}{Z} r_{uu}(0)$$

Leistung

$$P_u(t_1, t_2) = \frac{1}{t_2 - t_1} W_u(t_1, t_2)$$

$$P_u(n_1, n_2) = \frac{1}{n_2 - n_1 + 1} W_u(n_1, n_2)$$

$$P_u = P_u(-\infty, \infty) = P_{u_T}(\tau, \tau + T)$$

Varianz

$$\begin{aligned} \sigma_u^2(t_1, t_2) &= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} (u(t) - m_u(t_1, t_2))^2 dt \\ &= Z P_u(t_1, t_2) - m_u(t_1, t_2)^2 \end{aligned}$$

$$\begin{aligned} \sigma_u^2(n_1, n_2) &= \frac{1}{n_2 - n_1 + 1} \sum_{v=n_1}^{n_2} (u(v) - m_u(n_1, n_2))^2 \\ &= Z P_u(n_1, n_2) - m_u(n_1, n_2)^2 \end{aligned}$$

$$\begin{aligned} \sigma_u^2 &= \sigma_u^2(-\infty, \infty) = \sigma_{u_T}^2(\tau, \tau + T) \\ &= Z P_u - m_u^2 \end{aligned}$$

Kreuzkorrelationsfunktion

$$r_{uv}(t_1, t_2)(t) = \int_{t_1}^{t_2} u(\tau)v(t + \tau) d\tau$$

$$r_{uv}(n_1, n_2)(n) = \sum_{v=n_1}^{n_2-n} u(v)v(n + v)$$

$$r_{uv}(t) = r_{uv}(-\infty, \infty)(t) = r_{u_T v_T}(\tau, \tau + T)(t)$$

normiert

$$\rho_{uv}(t) = \frac{r_{uv}(t)}{r_{uv}(0)}$$

Leistungssignale

$${}^L r_{uv}(t) = \lim_{T \rightarrow \infty} \left(\frac{1}{T} \int_{-T}^T u(\tau)v(t + \tau) d\tau \right)$$

Signal-Verhältnis

$$SR(u, v) = \frac{P_u}{P_v} = \frac{U_{\text{eff}}^2}{V_{\text{eff}}^2}$$

Signal-Abstand

$$SR_{\text{dB}}(u, v) = \log_{10} \left(\frac{P_u}{P_v} \right) [\text{B}] = 2 \log_{10} \left(\frac{U_{\text{eff}}}{V_{\text{eff}}} \right) [\text{B}]$$

Hilberttransformation

$$\mathcal{H}(u(t)) = u(t) \star \frac{1}{t} = \int_{-\infty}^{\infty} \frac{u(\tau)}{t - \tau} d\tau$$

$$\mathcal{H}(\mathcal{H}(u(t))) = u(t) \star \frac{1}{t} \star \frac{1}{t} = -\pi^2 u(t)$$

Digitalisierung

Ideale Abtastung

$$u^*(t) = u(t)\delta_T(t) = \sum_{n=-\infty}^{\infty} u(nT)\delta(t-nT) \quad \circ \rightarrow \quad U^*(\omega) = \frac{1}{2\pi} U(\omega) \star \omega_T \delta_{\omega_T}(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} U(\omega - k\omega_T)$$

$$u(t) = u^*(t) \star T h_{\text{TP}}\left(\frac{\omega_T}{2}\right)(t) = \sum_{n=-\infty}^{\infty} u(nT) \text{si}\left(\frac{\omega_T}{2}(t-nT)\right) \quad \circ \rightarrow \quad U(\omega) = U^*(\omega) T H_{\text{TP}}\left(\frac{\omega_T}{2}\right)(\omega)$$

Signalausblendung (shape-top sampling, natural sampling) mit $0 < \alpha \leq 1$

$$u_a(t) = u(t) (\Pi_{\alpha T}(t) \star \delta_T(t)) \quad \circ \rightarrow \quad U_a(\omega) = \frac{1}{2\pi} U(\omega) \star \alpha T \text{si}\left(\frac{\omega \alpha T}{2}\right) \omega_T \delta_{\omega_T}(\omega) = \alpha \sum_{k=-\infty}^{\infty} \text{si}(k\pi\alpha) U(\omega - k\omega_T)$$

$$u(t) = u_a(t) \star \frac{1}{\alpha} h_{\text{TP}}\left(\frac{\omega_T}{2}\right)(t) \quad \circ \rightarrow \quad U(\omega) = U_a(\omega) \frac{1}{\alpha} H_{\text{TP}}\left(\frac{\omega_T}{2}\right)(\omega)$$

Signalverbreiterung (flat-top sampling)

$$u_a(t) = u^*(t) \star \Pi_{\alpha T}(t) \quad \circ \rightarrow \quad U_a(\omega) = \alpha \text{si}\left(\frac{\omega \alpha T}{2}\right) \sum_{k=-\infty}^{\infty} U(\omega - k\omega_T)$$

Abtastrate Bandpaßsignale

$$f_T \geq 2f_g \quad f_T = \frac{2}{n} f_{g\text{max}} \quad \text{mit } n \in \mathbb{N} \wedge \frac{f_{g\text{max}}}{f_B} - 1 < n \leq \frac{f_{g\text{max}}}{f_B}$$

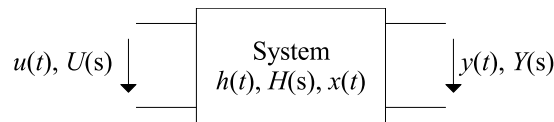
Quantisierungsrauschen $q(t)$ (Quantisierungsstufenhöhe Δ , Bitanzahl m)

$$P_q = \frac{1}{Z} \frac{\Delta^2}{12} = \frac{1}{Z} \frac{\hat{u}^2}{2^{2m} \cdot 3} \quad \text{SNR}(u) = \text{SR}(u, q) = \alpha 2^{2m} \quad \text{SNR}_{\text{dB}}(u) = 0,602m + \log_{10}(\alpha) [\text{B}]$$

Spezielle Systeme

Bezeichnung	Definition/Eigenschaften	bei Pol-/Nullstellen	bei diskreten PN
Stabil	$W_h < \infty$	$\Re(s_{\infty}) < 0$	$ z_{\infty} < 1$
Kausal	$h(t < 0) = 0$ $\int_{-\infty}^{\infty} \frac{ \ln(A(\omega)) }{1 + \frac{\omega^2}{\omega_B^2}} d\omega < \infty$ $h(t) = h_g(t) + h_u(t) \quad \circ \rightarrow \quad H(\omega) = H_g(\omega) + H_u(\omega)$ $h_g(t) = h_u(t) \text{sgn}(t) \quad \circ \rightarrow \quad H_g(\omega) = \frac{1}{i\pi} \mathcal{H}(H_u(\omega))$	$R \leq Q$ (reale Systeme immer)	$R \leq Q$
Linearphasig (Stabil)	$H_{\text{LP}}(\omega) = A(\omega) \exp(-i\omega t_0) \quad \varphi_{\text{LP}}(\omega) = -\omega t_0$ $h(t_0 + t) = h(t_0 - t) \quad \vee \quad h(t_0 + t) = -h(t_0 - t)$	$\Re(s_0) = -\Re(s_0) \quad Q=0$ (reale Systeme nie)	$z_0 = \overline{z_0^{-1}} \quad Q=0$ $t_0 = TR/2$
Allpaß (Stabil)	$H_{\text{AP}}(\omega) = A_0 \exp(i\varphi(\omega)) \quad A_{\text{AP}}(\omega) = A_0$	$\Re(s_0) = -\Re(s_{\infty}) \quad R=Q$ $A_0 = \left \frac{b_R}{a_0} \right = \left \frac{b_0}{a_0} \right $	$z_0 = \overline{z_{\infty}^{-1}} \quad R=Q$ $A_0 = \left \frac{b_0}{a_0} \right \prod_{r=1}^R z_{0r} $ $= \left \frac{b_0}{a_0} \right \prod_{q=1}^Q z_{\infty q}^{-1} $ $b_r = H_{\text{AP}} a_{R-r}$
Verzerrungsfrei (Linearphasig und Allpaß)	$y_{\text{VS}}(t) = \alpha u(t - t_0) \quad H_{\text{VS}}(\omega) = A_0 \exp(-i\omega t_0)$		
Minimalphasig/ Allpaßfrei (Kausal)	$H_{\text{min}}(\omega) = A(\omega) \exp(i\varphi_{\text{min}}(\omega))$ $\varphi_{\text{min}}(\omega) = \mathcal{H}(\ln(A(\omega)))$	$\Re(s_{0 \infty}) < 0$	$ z_{0 \infty} < 1$
Nichtrekursiv	$Q = 0$ $h(n) = \frac{1}{a_0} \sum_{r=0}^R b_r \delta(n-r) \quad \circ \rightarrow \quad H(z) = \frac{1}{a_0} \sum_{r=0}^R b_r z^{-r} = \frac{b_0}{a_0} \frac{\prod_{r=1}^R (z-z_{0r})}{z^R}$		
Reinrekursiv	$R = 0$ $h(n) \quad \circ \rightarrow \quad H(z) = \frac{b_0}{\sum_{q=0}^Q a_q z^{-q}} = \frac{b_0}{a_0} \frac{z^Q}{\prod_{q=1}^Q (z-z_{\infty q})}$		

Systembeschreibung (Vierpol)



Vierpolparameter

$$\begin{aligned}
 \text{H-Parameter} \quad \begin{pmatrix} u_e \\ i_a \end{pmatrix} &= \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} \cdot \begin{pmatrix} i_e \\ u_a \end{pmatrix} & \quad \mathbf{H} &= \frac{1}{y_{11}} \begin{pmatrix} 1 & -y_{12} \\ y_{21} & |\mathbf{Y}| \end{pmatrix} = \frac{1}{z_{22}} \begin{pmatrix} |\mathbf{Z}| & z_{12} \\ -z_{21} & 1 \end{pmatrix} \\
 \text{Z-Parameter} \quad \begin{pmatrix} u_e \\ u_a \end{pmatrix} &= \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \cdot \begin{pmatrix} i_e \\ i_a \end{pmatrix} & \quad \mathbf{Z} &= \frac{1}{h_{22}} \begin{pmatrix} |\mathbf{H}| & h_{12} \\ -h_{21} & 1 \end{pmatrix} = \frac{1}{|\mathbf{Y}|} \begin{pmatrix} y_{22} & -y_{12} \\ -y_{21} & y_{11} \end{pmatrix} \\
 \text{Y-Parameter} \quad \begin{pmatrix} i_e \\ i_a \end{pmatrix} &= \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \cdot \begin{pmatrix} u_e \\ u_a \end{pmatrix} & \quad \mathbf{Y} &= \frac{1}{h_{11}} \begin{pmatrix} 1 & -h_{12} \\ h_{21} & |\mathbf{H}| \end{pmatrix} = \frac{1}{|\mathbf{Z}|} \begin{pmatrix} z_{22} & -z_{12} \\ -z_{21} & z_{11} \end{pmatrix}
 \end{aligned}$$



LTI-System (Linear Time Invariant)

Frequenzgang

$$H(\omega) = \frac{Y(\omega)}{U(\omega)} = A_h(\omega) \exp(i\varphi_h(\omega)) = H(s)|_{s=i\omega} = H(z)|_{z=\exp(i\omega T)}$$

Amplitudengang

Dämpfung=–Verstärkung

$$A_h(\omega) = |H(\omega)|$$

$$a_h(\omega) = \text{SR}_{\text{dB}}(U, Y) = -2 \log_{10}(|H(\omega)|) [\text{B}] = -v_h(\omega)$$

Phasengang

Phasenlaufzeit

$$\varphi_h(\omega) = \arg(H(\omega)) = -\omega t_{\text{phase}}(\omega) \quad t_{\text{phase}}(\omega) = -\frac{\varphi_h(\omega)}{\omega}$$

Systemantworten

$$u(t) \rightarrow y(t) = u(t) \star h(t) \quad \Leftrightarrow \quad Y(s) = U(s)H(s) \quad u(n) \rightarrow y(n) = u(n) \star h(n) \quad \Leftrightarrow \quad Y(z) = U(z)H(z)$$

Impulsantwort

$$\delta(t) \rightarrow h(t) = \frac{dh_\sigma(t)}{dt} \quad \Leftrightarrow \quad H(s) = sH_\sigma(s) \quad \delta(n) \rightarrow h(n) = h_\sigma(n) - h_\sigma(n-1) \quad \Leftrightarrow \quad H(z) = (1 - z^{-1})H_\sigma(z)$$

Sprungantwort

$$\sigma(t) \rightarrow h_\sigma(t) = \int_{-\infty}^t h(\tau) d\tau \quad \Leftrightarrow \quad H_\sigma(s) = s^{-1}H(s) \quad \sigma(n) \rightarrow h_\sigma(n) = \sum_{v=-\infty}^n h(n) \quad \Leftrightarrow \quad H_\sigma(z) = (1 - z^{-1})^{-1}H(z)$$

Exponentialsignale (unendlich ausgedehnt bzw. im eingeschwungenen Zustand)

$$\begin{aligned}
 u(t) = u_0 \exp(i\omega_1 t) &\rightarrow y(t) = u(t)H(\omega_1) & u(n) = u_0 \exp(i\omega_1 nT) &\rightarrow y(n) = u(n)H(\omega_1) \\
 \Rightarrow H(\omega) = \frac{y(t)}{u(t)} = \frac{Z_y}{Z_u} & & \Rightarrow H(\omega) = \frac{y(n)}{u(n)} &
 \end{aligned}$$

Pol-Nullstellen-Darstellung (Laplace-/Z-Transformation)

DGL

$$\sum_{q=0}^Q a_q \frac{d^q y(t)}{dt^q} = \sum_{r=0}^R b_r \frac{d^r u(t)}{dt^r} \quad \sum_{q=0}^Q a_q y(n-q) = \sum_{r=0}^R b_r u(n-r)$$

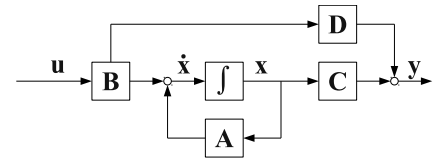
Übertragungsfunktion

$$H(s) = \frac{Y(s)}{U(s)} = \frac{Z(s) = \sum_{r=0}^R b_r s^r}{N(s) = \sum_{q=0}^Q a_q s^q} = \frac{b_R \prod_{r=1}^R (s-s_{0r})}{a_Q \prod_{q=1}^Q (s-s_{\infty r})} \quad H(z) = \frac{Y(z)}{U(z)} = \frac{\sum_{r=0}^R b_r z^{-r}}{\sum_{q=0}^Q a_q z^{-q}} = z^{Q-R} \frac{b_0 \prod_{r=1}^R (z-z_{0r})}{a_0 \prod_{q=1}^Q (z-z_{\infty r})}$$

Zustandsmodell

$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t), t)$ \mathbf{x} Zustandsvektor \mathbf{u} Eingangsgrößen

$y(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{u}(t), t)$ y Ausgangsgrößen



Linearisierung im Arbeitspunkt AP

$\dot{\mathbf{x}} \approx \mathbf{f}(\mathbf{x}_{AP}, \mathbf{u}_{AP}) + \mathbf{A} \cdot (\mathbf{x} - \mathbf{x}_{AP}) + \mathbf{B} \cdot (\mathbf{u} - \mathbf{u}_{AP})$ $\mathbf{A} = \frac{\partial \mathbf{f}}{\partial \mathbf{x}^T}(\mathbf{x}_{AP}, \mathbf{u}_{AP})$ $\mathbf{B} = \frac{\partial \mathbf{f}}{\partial \mathbf{u}^T}(\mathbf{x}_{AP}, \mathbf{u}_{AP})$

$y \approx \mathbf{g}(\mathbf{x}_{AP}, \mathbf{u}_{AP}) + \mathbf{C} \cdot (\mathbf{x} - \mathbf{x}_{AP}) + \mathbf{D} \cdot (\mathbf{u} - \mathbf{u}_{AP})$ $\mathbf{C} = \frac{\partial \mathbf{g}}{\partial \mathbf{x}^T}(\mathbf{x}_{AP}, \mathbf{u}_{AP})$ $\mathbf{D} = \frac{\partial \mathbf{g}}{\partial \mathbf{u}^T}(\mathbf{x}_{AP}, \mathbf{u}_{AP})$

$\mathbf{h}(t) = \mathbf{C} \cdot \Psi(t) \cdot \mathbf{B} + \mathbf{D}\delta(t)$ $\longleftrightarrow \mathbf{H}(s) = \mathbf{C} \cdot (s\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D}$ Impulsantwort

$\mathbf{h}_\sigma(t) = \int_0^t \mathbf{C} \cdot \Psi(\tau) \cdot \mathbf{B} d\tau + \mathbf{D} = \mathbf{C} \cdot \mathbf{A}^{-1} \Psi(t) \cdot \mathbf{B} - \mathbf{C} \cdot \mathbf{A}^{-1} \cdot \mathbf{B} + \mathbf{D}\sigma(t)$ $\longleftrightarrow \mathbf{H}_\sigma(s) = s^{-1} \mathbf{C} \cdot (s\mathbf{I} - \mathbf{A})^{-1} \cdot \mathbf{B} + \mathbf{D}$ Sprungantwort

$\mathbf{x}(t) = \Psi(t) \cdot \mathbf{x}(0) + \int_0^t \Psi(t - \tau) \cdot \mathbf{B} \cdot \mathbf{u}(\tau) d\tau$ $\Psi(t) = \exp(\mathbf{A}t)$ Bewegungsgleichung

$y(t) = \mathbf{C} \cdot \Psi(t) \cdot \mathbf{x}(0) + \int_0^t \mathbf{C} \cdot \Psi(t - \tau) \cdot \mathbf{B} \cdot \mathbf{u}(\tau) d\tau + \mathbf{D} \cdot \mathbf{u}(t)$ Ausgangsgleichung

$\mathbf{K}_s = -\mathbf{C} \cdot \mathbf{A}^{-1} \cdot \mathbf{B} + \mathbf{D}$ statische Verstärkung

Regelungsnormalform ($R \leq Q$)

$\mathbf{x} = \frac{a_q}{b_0} \begin{pmatrix} y \\ \vdots \\ \frac{d^{Q-1}y}{dt^{Q-1}} \end{pmatrix}$ $\mathbf{A} = \begin{pmatrix} 0 & 1 & & 0 \\ \vdots & & \ddots & \\ 0 & 0 & & 1 \\ -\frac{a_0}{a_Q} & -\frac{a_1}{a_Q} & \dots & -\frac{a_{Q-1}}{a_Q} \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$ $\mathbf{C} = \left(\frac{b_0}{a_Q} - \frac{b_Q a_0}{a_Q^2} \quad \dots \quad \frac{b_{Q-1}}{a_Q} - \frac{b_Q a_{Q-1}}{a_Q^2} \right)$ $\mathbf{D} = \frac{b_Q}{a_Q}$ $\mathbf{K}_s = \frac{b_0}{a_0}$

Beobachtungsnormalform ($R \leq Q$)

$\mathbf{A} = \begin{pmatrix} 0 & \dots & 0 & -a_0/a_Q \\ 1 & & 0 & -a_1/a_Q \\ & \ddots & & \vdots \\ 0 & & 1 & -a_{Q-1}/a_Q \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} b_0/a_Q - b_n a_0/a_Q^2 \\ \vdots \\ b_{Q-1}/a_Q - b_n a_{Q-1}/a_Q^2 \end{pmatrix}$ $\mathbf{C} = (0 \quad \dots \quad 0 \quad 1)$ $\mathbf{D} = \frac{b_Q}{a_Q}$ $\mathbf{K}_s = \frac{b_0}{a_0}$

Kanonische Normalform (λ_q Eigenwerte, λ_q Eigenvektoren von \mathbf{A})

$\mathbf{x}' = \mathbf{V}^{-1} \cdot \mathbf{x}$ $\mathbf{A}' = \mathbf{V}^{-1} \cdot \mathbf{A} \cdot \mathbf{V} = \text{diag}(\lambda_q)$ $\mathbf{B}' = \mathbf{V}^{-1} \cdot \mathbf{B}$ $\mathbf{C}' = \mathbf{C} \cdot \mathbf{V}$ $\mathbf{D}' = \mathbf{D}$ $\mathbf{V} = (\lambda_1 \quad \dots \quad \lambda_Q)$

Eigenbewegung

Eigenlösung

$\Psi'(t) = \text{diag}(\exp(\lambda_q t))$ $\Psi'(t) \cdot \mathbf{x}'(0) = \begin{pmatrix} \exp(\lambda_1 t) x'_1 \\ \vdots \\ \exp(\lambda_Q t) x'_Q \end{pmatrix}$ $\Psi(t) \cdot \mathbf{x}(0) = \sum_{q=1}^Q \lambda_q \exp(\lambda_q t) x'_q(0)$ $\mathbf{x}_q(t) = \lambda_q \exp(\lambda_q t)$

Stabilität

Ljapunow

stabil_{Ljapunow} $\Leftrightarrow \forall \varepsilon > 0 : \exists \delta > 0 : \forall \|\mathbf{x}(0)\| < \delta : \forall t > 0 : \|\mathbf{x}(t)\| < \varepsilon \Leftrightarrow \forall \lambda_q : \Re(\lambda_q) \leq 0$

asymptotisch stabil_{Ljapunow} \Leftrightarrow stabil_{Ljapunow} $\wedge \lim_{t \rightarrow \infty} (\|\mathbf{x}(t)\|) = 0 \Leftrightarrow \forall \lambda_q : \Re(\lambda_q) < 0$

Hurwitz (asymptotisch stabil)

$\det(\mathbf{A} - \lambda \mathbf{I}) = \sum_{i=0}^n a_i \lambda^i = 0$ $D_0 = a_1$ $D_1 = \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}$ $D_2 = \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix} \dots$

stabil_{Hurwitz} $\Leftrightarrow (\forall a_i | 0 \leq i \leq n : a_i > 0) \wedge (\forall D_i | 0 \leq i \leq n : D_i > 0)$

Routh (asymptotisch stabil)

zeile 1 $\hat{=} a_{n-0} \quad a_{n-2} \quad a_{n-4} \quad \dots \quad 0$ zeile j $\hat{=} z_{j,1} \quad z_{j,2} \quad \dots$

zeile 2 $\hat{=} a_{n-1} \quad a_{n-3} \quad a_{n-5} \quad \dots \quad 0$ $z_{j,1} = z_{j-2,i+1} - t_j z_{j-1,i+1}$ $t_j = \frac{z_{j-2,1}}{z_{j-1,1}}$

stabil_{Routh} $\Leftrightarrow (\forall a_i | 0 \leq i \leq n : a_i > 0) \wedge (\forall z_{j,1} : z_{j,1} > 0)$

BIBO (Bounded Input Bounded Output)

$\int_0^\infty |h(t)| dt < \infty \Leftrightarrow \forall s_{\infty q} : \Re(s_{\infty q}) < 0 \Leftrightarrow$ asymptotisch stabil

Betrachtung der Schleifenverstärkung H_0 bei Einheitsrückführung ($Q > R$)

Nyquistkriterium

$\forall \omega : H_0(i\omega) \neq \pm 1 \wedge \arg(1 \mp H_0(i\infty)) - \arg(1 \mp H_0(i0)) = n_r \pi + n_a \frac{\pi}{2}$ $n_r = \#(s_{\infty q} | \Re(s_{\infty q}) > 0)$ $n_a = \#(s_{\infty q} | \Re(s_{\infty q}) = 0)$

Phasenrand

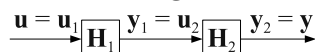
$\Phi_R > 0$ $\Phi_R = \left(\frac{3}{2} \pm \frac{1}{2}\right) \pi + \arg(H_0(i\omega_c))$ $|H_0(i\omega_c)| = 1$

Amplitudenrand

$k_R > 1$ $k_R = |H_0(i\omega_{-2\pi})|^{-1}$ $\arg(H_0(i\omega_{-2\pi})) - \left(\frac{1}{2} \mp \frac{1}{2}\right) \pi = -2\pi$

Gekoppelte Systeme

Reihenschaltung



$$A_h = A_{h_1} A_{h_2} \quad \phi_h = \phi_{h_1} + \phi_{h_2}$$

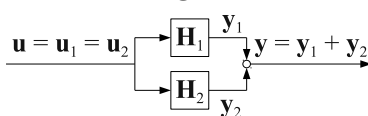
$$a_h = a_{h_1} + a_{h_2} \quad v_h = v_{h_1} + v_{h_2}$$

$$A = \begin{pmatrix} A_1 & \mathbf{0} \\ B_2 \cdot C_1 & A_2 \end{pmatrix} \quad B = \begin{pmatrix} B_1 \\ B_2 \cdot D_1 \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad A = \begin{pmatrix} A_1 & \mathbf{0} \\ \mathbf{0} & A_2 \end{pmatrix} \quad B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

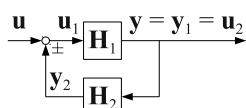
$$C = \begin{pmatrix} D_2 \cdot C_1 & C_2 \end{pmatrix} \quad D = \begin{pmatrix} D_2 \cdot D_1 \end{pmatrix} \quad C = \begin{pmatrix} C_1 & C_2 \end{pmatrix} \quad D = D_1 + D_2$$

$$h = h_2 \star h_1 \quad H = H_2 \cdot H_1 \quad h = h_1 + h_2 \quad H = H_1 + H_2$$

Parallelschaltung



Rückkopplung



$$H_0 = H_2 \cdot H_1 \quad \text{Schleifenübertragungsfunktion}$$

$$H = (1 \mp H_1 \cdot H_2)^{-1} \cdot H_1$$

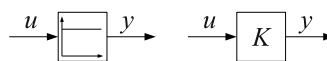
$$A = \begin{pmatrix} A_1 \pm B_1 \cdot D_2 \cdot (1 \mp D_1 \cdot D_2)^{-1} \cdot C_1 & \pm B_1 \cdot C_2 + B_1 \cdot D_2 \cdot (1 \mp D_1 \cdot D_2)^{-1} \cdot D_1 \cdot C_2 \\ B_2 \cdot (1 \mp D_1 \cdot D_2)^{-1} \cdot C_1 & A_2 \pm B_2 \cdot (1 \mp D_1 \cdot D_2)^{-1} \cdot D_1 \cdot C_2 \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$B = \begin{pmatrix} B_1 \pm B_1 \cdot D_2 \cdot (1 \mp D_1 \cdot D_2)^{-1} \cdot D_1 \\ B_2 \cdot (1 \mp D_1 \cdot D_2)^{-1} \cdot D_1 \end{pmatrix} \quad C = \begin{pmatrix} (1 \mp D_1 \cdot D_2)^{-1} \cdot C_1 & \pm (1 \mp D_1 \cdot D_2)^{-1} \cdot D_1 \cdot C_2 \end{pmatrix} \quad D = (1 \mp D_1 \cdot D_2)^{-1} \cdot D_1$$

Bausteine

P Proportionalblock

$$y = K_s u \quad h_\sigma(t) = K_s \sigma(t) \quad H(s) = K_s$$

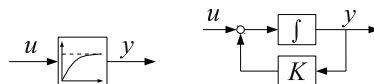


PT_n proportional im eingeschwungenen Zustand, n Zeitkonstanten

$$\sum_{q=1}^Q T_q \frac{d^q y}{dt^q} + y = K_s u \quad H(s) = \frac{K_s}{\sum_{q=1}^Q T_q s^q + 1}$$

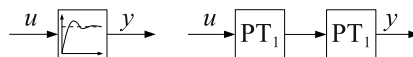
PT₁ T = Zeitkonstante

$$T \dot{y} + y = K_s u \quad h_\sigma(t) = K_s \left(1 - \exp\left(-\frac{t}{T}\right)\right) \sigma(t)$$



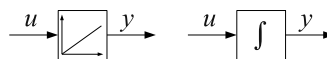
PT₂ d = Dämpfung

$$T^2 \ddot{y} + 2dT \dot{y} + y = K_s u$$



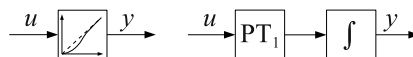
I Integralblock

$$T_I \dot{y} = u \quad h_\sigma(t) = \frac{t}{T_I} \sigma(t) \quad H(s) = \frac{1}{T_I s}$$



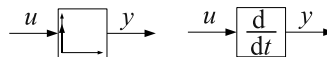
IT₁ (PT₁ + I)

$$T_I T \ddot{y} + T_I \dot{y} = u \quad h_\sigma(t) = \left(\frac{t}{T_I} - \frac{T}{T_I} \left(1 - \exp\left(-\frac{t}{T}\right)\right)\right) \sigma(t)$$



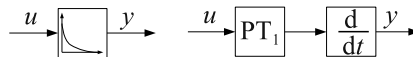
D Differentialblock

$$y = T_D \dot{u} \quad h_\sigma(t) = T_D \delta(t) \quad H(s) = s T_D$$



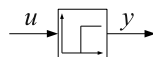
DT₁ (PT₁ + D)

$$T \dot{y} + y = T_D \dot{u}$$



T_t Totzeitblock

$$y(t) = K_s u(t - T_t) \quad h_\sigma(t) = K_s \sigma(t - T_t) \quad H(s) = K_s \exp(-s T_t)$$



Filter

3 dB-Grenzfrequenz äquivalente Bandbreite Transitfrequenz

$$A_h(\omega_g) = \frac{1}{\sqrt{2}} A_h(\omega_0) \quad A_h(\omega_0) \omega_{eq} = \int_{-\infty}^{\infty} A_h(\omega) d\omega \quad A_h(\omega_i) = 1$$

Tiefpaß

$$h_{TP}(\omega_g)(t) = \frac{\omega_g}{\pi} \text{si}(\omega_g(t - t_0))$$

$$h_{TP}(\omega_g)(n) = \frac{\omega_g}{\pi} \text{si}(\omega_g(n - n_0))$$

$$H_{TP}(\omega_g)(\omega) = \Pi_{2\omega_g}(\omega) \exp(-i\omega t_0)$$

$$H_{TP}(\omega_g)(\omega) = \Pi_{2\omega_g}(\omega) \exp(-i\omega n_0 T)$$

Bandpaß (Mittenfrequenz $\omega_m = (\omega_{g \min} + \omega_{g \max})/2$, Bandbreite $\omega_B = \omega_{g \max} - \omega_{g \min}$)

$$h_{BP}(\omega_B)(t) = 2h_{TP}\left(\frac{\omega_B}{2}\right)(t) \cos(\omega_m t)$$

$$h_{BP}(\omega_B)(n) = 2h_{TP}\left(\frac{\omega_B}{2}\right)(n) \cos(n\omega_m T)$$

$$H_{BP}(\omega_B)(\omega) = H_{TP}\left(\frac{\omega_B}{2}\right)(\omega - \omega_m) + H_{TP}\left(\frac{\omega_B}{2}\right)(\omega + \omega_m)$$

$$H_{BP}(\omega_B)(\omega) = H_{TP}\left(\frac{\omega_B}{2}\right)(\omega - \omega_m) + H_{TP}\left(\frac{\omega_B}{2}\right)(\omega + \omega_m)$$

Hochpaß

$$h_{HP}(\omega_g)(t) = \delta(t - t_0) - \frac{\omega_g}{\pi} \text{si}(\omega_g(t - t_0))$$

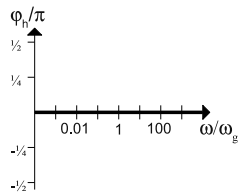
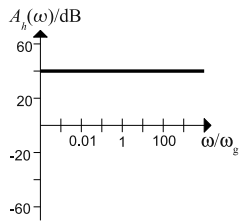
$$h_{HP}(\omega_B)(n) = h_{TP}(\omega_B)(n) \cos(n\pi)$$

$$H_{HP}(\omega_g)(\omega) = (1 - \Pi_{2\omega_g}(\omega)) \exp(-i\omega t_0)$$

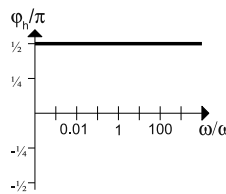
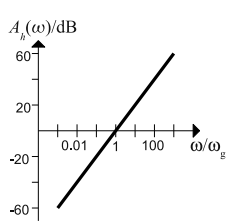
$$H_{HP}(\omega_B)(\omega) = H_{TP}(\omega_B)(\omega \pm \frac{\pi}{T})$$

Bode-Diagramm

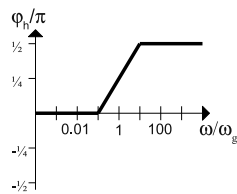
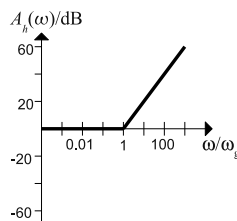
$H = \text{const}$



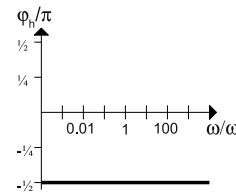
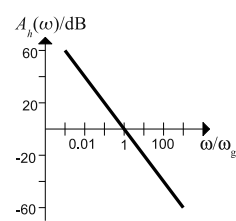
$H = i\omega/\omega_0$



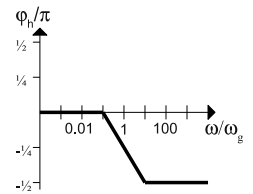
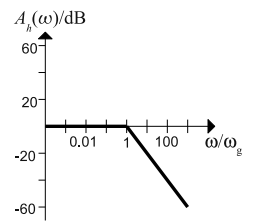
$H = 1 + i\omega/\omega_0$



$H = \frac{1}{i\omega/\omega_0}$



$H = \frac{1}{1 + i\omega/\omega_0}$



Allgemeines Filter

$$H(s) = \frac{H_0}{1 + \sum_{n=1}^N c_n s^n} \quad c_n \in \mathbb{R}_0^+$$

Tiefpaß

$$S' = S$$

Hochpaß

$$S' = \frac{1}{S}$$

$$S = \frac{s}{\omega_g}$$

Bandpaß

$$S' = \frac{1}{\Delta\Omega} \left(S + \frac{1}{S} \right)$$

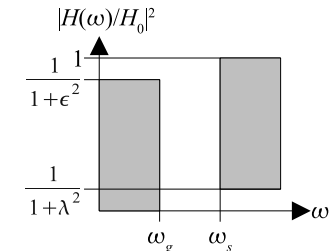
Bandsperr

$$S' = \frac{\Delta\Omega}{S + \frac{1}{S}}$$

$$S = \frac{s}{\omega_0}$$

$$\Delta\Omega = \frac{\omega_{g2} - \omega_{g1}}{\omega_0}$$

Filterentwurf



Butterworth-Approximation

$$|H(\omega)| = \frac{|H_0|}{\sqrt{1 + \epsilon^2 \Omega^{2N}}} \quad s_{\infty n} = \frac{\omega_g}{\sqrt[n]{\epsilon}} \exp\left(i \frac{\pi}{2} \left(1 + \frac{2n-1}{N}\right)\right)$$

$$N \geq \frac{\log\left(\frac{1}{\epsilon}\right)}{\log\left(\frac{\omega_s}{\omega_g}\right)}$$

Tschebyscheff-Approximation

$$|H(\omega)| = \frac{|H_0|}{\sqrt{1 + \epsilon^2 T_N(\Omega)^2}} \quad T_N(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

$$T_0(x) = 1 \quad T_1(x) = x$$

$$N \geq \frac{\arccos\left(\frac{1}{\epsilon}\right)}{\arccos\left(\frac{\omega_s}{\omega_g}\right)}$$

Cauer-Approximation

$$|H(\omega)| = \frac{|H_0|}{\sqrt{1 + \epsilon^2 \Psi_N(\Omega)^2}} \quad \Psi(x) \text{ elliptische Jacobi-Funktion}$$

Bessel-Approximation

$$H(s) = H_0 \frac{B_N(0)}{B_N(sT_{\text{phase}})} \quad B_N(x) = (2n-1)B_{n-1}(x) + B_{n-2}(x)$$

$$B_0(x) = 1 \quad B_1(x) = x + 1$$