

Elektrisches Feld

$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt} \quad \oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}$$

$$\nabla \cdot \mathbf{D} = \rho_V \quad \oint \mathbf{D} \cdot d\mathbf{A} = \int \rho_V dV$$

$$\mathbf{F} = q\mathbf{E} = (\mathbf{p}_e \cdot \nabla) \mathbf{E} \quad \mathbf{T} = \mathbf{p}_e \times \mathbf{E}$$

$$W = \frac{1}{2}CU^2 = \frac{1}{2}QU = -\mathbf{p}_e \cdot \mathbf{E} \quad w_V = \frac{1}{2}\mathbf{D} \cdot \mathbf{E} = \frac{1}{2}\rho_V\phi$$

$$P = UI$$

$$\mathbf{D} = \frac{dQ}{dA} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon\mathbf{E} \quad \text{Verschiebungsdichte/Erregung}$$

$$\mathbf{E} = \frac{dU}{ds} = \gamma^{-1}\mathbf{J} = \epsilon^{-1}\mathbf{D} = \epsilon_0^{-1}(\mathbf{D} - \mathbf{P}) = -\nabla\phi - \frac{\partial\mathcal{A}}{\partial t} = \frac{d\mathbf{F}}{dq} \quad \text{Feldstärke}$$

$$\mathbf{P} = n\mathbf{p}_e = \mathbf{D} - \epsilon_0\mathbf{E} = \epsilon_0\chi_e\mathbf{E} \quad \text{Polarisation}$$

$$\phi = \int_{r_0}^r \mathbf{E} \cdot d\mathbf{s} = 2\frac{w_V}{\rho_V} \quad \text{Potential}$$

$$\epsilon = \epsilon_r\epsilon_0 = (1 + \chi_e)\epsilon_0 = \frac{D}{E} \quad \text{Dielektrizität}$$

$$\gamma = \frac{1}{\rho} = \frac{J}{E} = nq\mu \quad \text{spezifischer Leitwert}$$

Magnetisches Feld

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{d\mathbf{D}}{dt} \quad \oint \mathbf{H} \cdot d\mathbf{s} = \int (\mathbf{J} + \frac{d\mathbf{D}}{dt}) \cdot d\mathbf{A}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \oint \mathbf{B} \cdot d\mathbf{A} = 0$$

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = (\mathbf{p}_m \cdot \nabla) \mathbf{B} \quad \mathbf{T} = \mathbf{p}_m \times \mathbf{B}$$

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\phi V = -\mathbf{p}_m \cdot \mathbf{B} \quad w_V = \frac{1}{2}\mathbf{B} \cdot \mathbf{H} = \frac{1}{2}\mathbf{J} \cdot \mathcal{A}$$

$$p_V = \mathbf{J} \cdot \mathbf{E}$$

$$\mathbf{B} = \frac{d\phi}{dA} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu\mathbf{H} = \nabla \times \mathcal{A} = \frac{\mathbf{F}}{q|\mathbf{v}|} \times \frac{\mathbf{v}}{|\mathbf{v}|} \quad \text{Flußdichte}$$

$$\mathbf{H} = \frac{dV}{ds} = \mu^{-1}\mathbf{B} = \mu_0^{-1}\mathbf{B} - \mathbf{M} \quad \text{Feldstärke/Erregung}$$

$$\mathbf{M} = n\mathbf{p}_m = \mu_0^{-1}\mathbf{B} - \mathbf{H} = \chi_m\mathbf{H} \quad \text{Magnetisierung}$$

$$\mathcal{A} \quad \text{Vektorpotential}$$

$$\mu = \mu_r\mu_0 = (1 + \chi_m)\mu_0 = \frac{B}{H} \quad \text{Permeabilität}$$

$$\rho_V = \frac{dQ}{dV} = \frac{J}{v}$$

$$\rho_A = \frac{dQ}{dA}$$

$$\mathbf{p}_e = Q\mathbf{d}$$

$$\rho_{V_{pol}} = -\nabla \cdot \mathbf{P}$$

$$\rho_{A_{pol}} = \frac{\mathbf{P} \cdot \mathbf{A}}{|\mathbf{A}|}$$

$$\mathbf{J} = \frac{dI}{dA} = \rho_V\mathbf{v} = \gamma\mathbf{E}$$

$$\mathbf{J}_A = \frac{dI}{ds}$$

$$\mathbf{p}_m = \pi a^2 \mathbf{e}_n$$

$$\mathbf{J}_{mag} = \nabla \times \mathbf{M}$$

$$\mathbf{J}_{A_{mag}} = \mathbf{M} \times \frac{\mathbf{A}}{|\mathbf{A}|}$$

$$U = \phi_2 - \phi_1 = \int_{r_1}^{r_2} \mathbf{E} \cdot d\mathbf{s} = -\frac{d\psi}{dt} = \pm L \frac{dI}{dt} = \frac{I}{Y}$$

$$I = \int \mathbf{J} \cdot d\mathbf{A} = \frac{dQ}{dt} = \frac{V}{N} = C \frac{dU}{dt} = UY$$

$$Q = \oint \mathbf{D} \cdot d\mathbf{A} = \oint Idt = CU \quad \text{Gaußscher Satz}$$

$$V = \int \mathbf{J} \cdot d\mathbf{A} = \oint \mathbf{H} \cdot d\mathbf{s} = NI = \frac{\phi}{Y_m} \quad \text{Durchflutung}$$

$$\phi = \int \mathbf{B} \cdot d\mathbf{A} = \oint \mathcal{A} \cdot d\mathbf{s} = \frac{1}{N} \int U dt = L \frac{I}{N} = VY_m \quad \text{Fluß}$$

$$\psi = N\phi = -\int U dt = LI \quad \text{verketteter Fluß}$$

$$C = \frac{Q}{U} = \epsilon \frac{A}{d} = 2 \frac{W}{U^2}$$

$$Y = \frac{1}{R} = \frac{I}{U} = \gamma \frac{A}{d} = i\omega C = \frac{1}{i\omega L}$$

Kapazität
Admittanz (Leitwert)

$$L = \frac{\psi}{I} = N^2 Y_m = 2 \frac{W}{I^2}$$

$$Y_m = \frac{1}{Z_m} = \frac{\phi}{V} = \mu \frac{A}{d}$$

$M = \frac{\mu}{4\pi} \iint \frac{1}{r} d\mathbf{s}_1 d\mathbf{s}_2$
magnetischer Leitwert

Beziehungen mit ε, γ, μ

$$\mathbf{D} = \varepsilon \mathbf{E} \quad \mathbf{J} = \gamma \mathbf{E} \quad \mathbf{B} = \mu \mathbf{H} \quad \mathbf{c} = \frac{1}{\sqrt{\varepsilon \mu}} \frac{\mathbf{k}}{|\mathbf{k}|} \quad Y = \sqrt{\frac{\varepsilon}{\mu}} = \left[\frac{H}{E} \right]_{\text{ebene Welle}} \quad \text{Relaxationszeit: } T_r = \frac{\varepsilon}{\gamma}$$

Wellengleichung ($\varepsilon, \gamma, \mu = \text{const}$) (gilt für $\mathbf{E}, \mathbf{D}, \mathbf{J}, \mathbf{H}, \mathbf{B}$)

$$\nabla^2 \mathbf{E} - \nabla \cdot (\nabla \cdot \mathbf{E}) = \gamma \mu \frac{\partial \mathbf{E}}{\partial t} + \varepsilon \mu \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad \mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t))$$

$$\text{Wellenzahl: } \mathbf{k}^2 = \omega^2 \mu \varepsilon \left(1 - i \frac{\gamma}{\omega \varepsilon}\right) \quad \text{Dämpfungskonstante: } \Im(k) \quad \text{Phasenkonstante: } \Re(k) = \frac{2\pi}{\lambda}$$

$$\text{Energiestrom } \mathbf{S} = \frac{dP}{dA} = \mathbf{E} \times \mathbf{H} = \mathbf{c} w_V \quad w_V = w_{V_e} + w_{V_m} \quad w_{V_e} = w_{V_m}$$

Kugelwelle

$$\begin{aligned} \phi &= \frac{\mathbf{r} \cdot \mathbf{p}_e}{4\pi \varepsilon r^3} (1 - i \mathbf{k} \cdot \mathbf{r}) \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t)) \\ \mathbf{E} &= \frac{1}{4\pi \varepsilon r^3} (\mathbf{r}(\mathbf{r} \cdot \mathbf{p}_e) (3 - i 3 \mathbf{k} \cdot \mathbf{r} - (\mathbf{k} \cdot \mathbf{r})^2) - r^2 \mathbf{p}_e (1 - i \mathbf{k} \cdot \mathbf{r} - (\mathbf{k} \cdot \mathbf{r})^2)) \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t)) \\ \mathcal{A} &= -i \frac{\mu \omega \mathbf{p}_e}{4\pi r} \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t)) \\ \mathbf{H} &= -i \frac{(\mathbf{p}_e \times \mathbf{r})}{4\pi r^3} (1 - i \mathbf{k} \cdot \mathbf{r}) \exp(i(\mathbf{k} \cdot \mathbf{r} - \omega t)) \end{aligned}$$

Potentialdifferentialgleichungen

Eichung

$$\begin{aligned} \nabla \cdot \mathcal{A} &= 0 & \nabla^2 \phi &= -\frac{\rho_V}{\varepsilon} & \nabla^2 \mathcal{A} &= -\mu \mathbf{J} \\ \nabla \cdot \mathcal{A} &= -\gamma \mu \phi & \nabla^2 \phi &= \gamma \mu \phi - \frac{\rho_V}{\varepsilon} & \nabla^2 \mathcal{A} &= \gamma \mu \dot{\mathcal{A}} \\ \nabla \cdot \mathcal{A} &= -\varepsilon \mu \dot{\phi} & \nabla^2 \phi &= \varepsilon \mu \ddot{\phi} - \frac{\rho_V}{\varepsilon} & \nabla^2 \mathcal{A} &= \varepsilon \mu \ddot{\mathcal{A}} - \mu \mathbf{J} \end{aligned}$$

Randbedingungen

Dielektrika	Leiter	Ferromagnetika
$E_{t1} = E_{t2}$	$E_{t1} = E_{t2}$	$H_{t1} = H_{t2} + J_F$
$D_{n1} = D_{n2} + \rho_F$	$J_{n1} = J_{n2}$	$B_{n1} = B_{n2}$
$\frac{\tan(\alpha_1)}{\tan(\alpha_2)} = \frac{E_{n2}}{E_{n1}} = \frac{D_{t1}}{D_{t2}} = \left(1 - \frac{\rho_F}{D_{n1}}\right) \frac{\varepsilon_1}{\varepsilon_2}$	$\frac{\tan(\alpha_1)}{\tan(\alpha_2)} = \frac{E_{n2}}{E_{n1}} = \frac{J_{t1}}{J_{t2}} = \frac{\gamma_1}{\gamma_2}$	$\frac{\tan(\alpha_1)}{\tan(\alpha_2)} = \frac{H_{n2}}{H_{n1}} = \frac{B_{t1}}{B_{t2}} = \left(1 + \frac{J_F}{H_{t2}}\right) \frac{\mu_1}{\mu_2}$

Felder ($\frac{q}{\varepsilon} \hat{=} \frac{I}{\gamma}$)

$$\begin{aligned} \phi &= \int \frac{1}{4\pi \varepsilon} \rho_V(\mathbf{r}') \left(\frac{1}{r} - \frac{1}{r_0}\right) dV' = \frac{1}{4\pi \varepsilon} Q \left(\frac{1}{r} - \frac{1}{r_0}\right) = \frac{1}{2\pi \varepsilon} \rho_L \ln\left(\frac{r_0}{r}\right) = \frac{1}{2\varepsilon} \rho_A (r_0 - r) = \frac{1}{4\pi \varepsilon} \mathbf{p}_e \cdot \left(\frac{\mathbf{r}}{r^3} - \frac{\mathbf{r}_0}{r_0^3}\right) \\ \mathbf{E} &= \int \frac{1}{4\pi \varepsilon} \rho_V(\mathbf{r}') \frac{\mathbf{r}}{r^3} dV' = \frac{1}{4\pi \varepsilon} Q \frac{\mathbf{r}}{r^3} = \frac{1}{2\pi \varepsilon} \rho_L \frac{\mathbf{r}}{r^2} = \frac{1}{2\varepsilon} \rho_A \frac{\mathbf{r}}{r} = \frac{1}{4\pi \varepsilon} p_e \frac{1}{r^3} (2 \cos(\vartheta) \mathbf{e}_r + \sin(\vartheta) \mathbf{e}_\phi) \\ \mathcal{A} &= \int \frac{\mu}{4\pi} \mathbf{J}(\mathbf{r}') \left(\frac{1}{r} - \frac{1}{r_0}\right) dV' = \int \frac{\mu}{3\pi} \mathbf{I}(\mathbf{r}') \left(\frac{1}{r} - \frac{1}{r_0}\right) ds' = \frac{\mu}{2\pi} \mathbf{I}_L \ln\left(\frac{r_0}{r}\right) = \frac{\mu}{2} \mathbf{I}_A (r_0 - r) = \frac{\mu}{4\pi} \mathbf{p}_m \times \left(\frac{\mathbf{r}}{r^3} - \frac{\mathbf{r}_0}{r_0^3}\right) \\ \mathbf{B} &= \int \frac{\mu}{4\pi} \mathbf{J}(\mathbf{r}') \times \frac{\mathbf{r}}{r^3} dV' = \int \frac{\mu}{4\pi} \mathbf{I}(\mathbf{r}') \times \frac{\mathbf{r}}{r^3} ds' = \frac{\mu}{2\pi} \mathbf{I}_L \times \frac{\mathbf{r}}{r^2} = \frac{\mu}{2} \mathbf{I}_A \times \frac{\mathbf{r}}{r} = \frac{\mu}{4\pi} p_m \frac{1}{r^3} (2 \cos(\vartheta) \mathbf{e}_r + \sin(\vartheta) \mathbf{e}_\phi) = \frac{\mu}{4\pi} Q \mathbf{v} \times \frac{\mathbf{r}}{r^3} \end{aligned}$$